7

Mechanical Properties of Solids

TOPIC 1 Hooke's Law & Young's Modulus

1. If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are: **[Sep. 06, 2020 (I)]**

(a) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$ (b) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$

(c) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{4B}$ (d) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

2. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is _______ .

[9 Jan 2020 I]

3. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to : **[12 April 2019 II]**

(a) $9F/(\pi r^2 YT)$ (b) $6F/(\pi r^2 YT$

(c) $3F/(\pi r^2 YT)$ (d) $F/(3\pi r^2 YT)$

4. In an environment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,

[Given, the Young's modulus for steel and brass are, respectively, $120\times10^9 N/m^2$ and $60\times10^9 N/m^2$]

[10 April 2019 II]

(a) 1.2×10^6 N/m^2 (b) 4.0×10^6 N/m^2

(c) $1.8\times10^6 N/m^2$ (d) $0.2\times10^6 N/m^2$

5. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

[10 April 2019 II]

(a) 1.00 mm (b) 1.16 mm

(c) 0.90 mm (d) 1.36 mm

6. A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given that g = 3.1 À ms^{-2}, what will be the tensile stress that would be developed in the wire? **[9 April 2019 I]**

(a) 6.2×10^6 Nm^{-2} (b) 5.2×10^6 Nm^{-2}

(c) 3.1×10^6 Nm^{-2} (d) 4.8×10^6 Nm^{-2}

7. A steel wire having a radius of 2.0 mm, carrying a load of 4kg, is hanging from a ceiling. Given that g = 3.1 À ms^{-2}, what will be the tensile stress that would be developed in the wire? **[8 April 2019 I]**

(a) $6.2 \times 10^6 Nm^{-2}$ (b) $5.2 \times 10^6 Nm^{-2}$

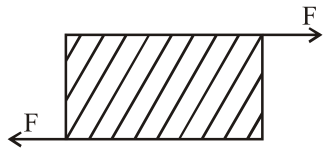
(c) $3.1 \times 10^6 Nm^{-2}$ (d) $4.8 \times 10^6 Nm^{-2}$

8. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :

[8 April 2019 II]

(a) 1.5mm (b) 1.9mm (c) 1.7mm (d) 1.3mm

9. As shown in the figure, forces of 10^5N each are applied in opposite directions, on the upper and lower faces of a cube of side 10cm, shifting the upper face parallel to itself by 0.5cm. If the side of another cube of the same material is, 20cm, then under similar conditions as above, the displacement will be: **[Online April 15, 2018]**



(a) 1.00cm (b) 0.25cm

(c) 0.37cm (d) 0.75cm

10. A thin 1 m long rod has a radius of 5 mm. A force of 50 πkN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false ? **[Online April 10, 2016]**

(a) The maximum value of Y that can be determined is 2×10^{14}N/m^2.

(b) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length

(c) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain

(d) The figure of merit is the largest for the length of the rod.

11. A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and 3R, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal : **[Online April 9, 2016]**

(a) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (b) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

(c) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (d) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

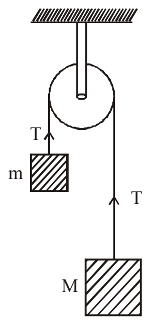
12. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is:

(For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1}) **[2014]**

(a) 2.2×10^{8} Pa (b) 2.2×10^{9} Pa

(c) 2.2×10^{7} Pa (d) 2.2×10^{6} Pa

13. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If M = 2 m, then the stress produced in the wire is : **[Online April 25, 2013]**



(a) $\frac{2mg}{3A}$ (b) $\frac{4mg}{3A}$ (c) $\frac{mg}{A}$ (d) $\frac{3mg}{4A}$

14. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's moduli of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is : **[Online April 23, 2013]**

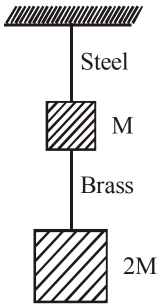
(a) 1.75 mm (b) 2.0 mm (c) 1.50 mm (d) 1.25 mm

15. A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to : **[Online April 22, 2013]**

(a) 1.0×10^{-4} (b) 1.5×10^{-4}

(c) 0.25×10^{-4} (d) 5×10^{-4}

16. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is : **[Online April 9, 2013]**



(a) $\frac{3c}{2ab^2}$ (b) $\frac{2a^2c}{b}$ (c) $\frac{3a}{2b^2c}$ (d) $\frac{2ac}{b^2}$

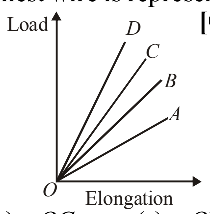
17. A steel wire can sustain 100 kg weight without breaking. If the wire is cut into two equal parts, each part can sustain a weight of **[Online May 19, 2012]**

(a) 50 kg (b) 400 kg (c) 100 kg (d) 200 kg

18. A structural steel rod has a radius of 10 mm and length of 1.0 m. A 100 kN force stretches it along its length. Young's modulus of structural steel is 2×10^{11} Nm^{-2}. The percentage strain is about **[Online May 7, 2012]**

(a) 0.16% (b) 0.32% (c) 0.08% (d) 0.24%

19. The load versus elongation graphs for four wires of same length and made of the same material are shown in the figure. The thinnest wire is represented by the line **[Online May 7, 2012]**



(a) OA (b) OC (c) OD (d) OB

20. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[2009]**

(a) $4F$ (b) $6F$ (c) $9F$ (d) F

21. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm) **[2006]**

(a) l (b) $2l$ (c) zero (d) $l/2$

TOPIC 2 Bulk and Rigidity Modulus and Work Done in Stretching a Wire

22. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is: **[9 Jan 2020 II]**

(a) $\sqrt{2}:1$ (b) 1 : 2
(c) 2 : 1 (d) $1:\sqrt{2}$

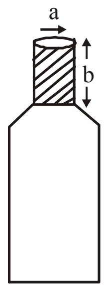
23. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to : **[8 April 2019 I]**

(a) $10^6 N/m^{-2}$ (b) $10^4 N/m^{-2}$
(c) $10^8 N/m^{-2}$ (d) $10^3 N/m^{-2}$

24. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is : **[2018]**

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$ (c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

25. A bottle has an opening of radius a and length b. A cork of length b and radius $(a + \Delta a)$ where $(\Delta a << a)$ is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is : **[Online April 10, 2016]**



(a) $(\pi\mu B\, b)\, a$ (b) $(2\pi\mu Bb)\, \Delta a$
(c) $(\pi\mu B\, b)\, \Delta a$ (d) $(4\, \pi\mu B\, b)\, \Delta a$

26. Steel ruptures when a shear of 3.5×10^8 N m^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly: **[Online April 12, 2014]**

(a) 1.4×10^4 N (b) 2.7×10^4 N
(c) 3.3×10^4 N (d) 1.1×10^4 N

27. The bulk moduli of ethanol, mercury and water are given as 0.9, 25 and 2.2 respectively in units of 10^9 Nm^{-2}. For a given value of pressure, the fractional compression in volume is $\frac{\Delta V}{V}$. Which of the following statements about $\frac{\Delta V}{V}$ for these three liquids is correct? **[Online April 11, 2014]**

(a) Ethanol > Water > Mercury
(b) Water > Ethanol > Mercury
(c) Mercury > Ethanol > Water
(d) Ethanol > Mercury > Water

28. In materials like aluminium and copper, the correct order of magnitude of various elastic modului is: **[Online April 9, 2014]**

(a) Young's modulus < shear modulus < bulk modulus.
(b) Bulk modulus < shear modulus < Young's modulus
(c) Shear modulus < Young's modulus < bulk modulus.
(d) Bulk modulus < Young's modulus < shear modulus.

29. If 'S' is stress and 'Y' is young's modulus of material of a wire, the energy stored in the wire per unit volume is **[2005]**

(a) $\frac{S^2}{2Y}$ (b) $2S^2Y$ (c) $\frac{S}{2Y}$ (d) $\frac{2Y}{S^2}$

30. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is **[2004]**

(a) $2F\ell$ (b) $F\ell$ (c) $\frac{F}{2\ell}$ (d) $\frac{F\ell}{2}$

Hints & Solutions

1. **(c)** Given : $U = \frac{-A}{r^6} + \frac{B}{r^{12}}$

For equilibrium,

$$F = \frac{dU}{dr} = -(A(-6r^{-7})) + B(-12r^{-13}) = 0$$

$$\Rightarrow 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}} \Rightarrow \frac{6A}{12B} = \frac{1}{r^6}$$

$\therefore$ Separation between molecules, $r = \left(\frac{2B}{A}\right)^{1/6}$

Potential energy,

$$U\left(r = \left(\frac{2B}{A}\right)^{1/6}\right) = -\frac{A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

2. **(4)** Given : Wire length, $l = 0.3$ m

Mass of the body, m = 10 kg

Breaking stress, $\sigma = 4.8 \times 10^7$ Nm^{-2}

Area of cross-section, a = 10^{-2} cm^2

Maximum angular speed $\omega = ?$

$T = Ml\omega^2$

$$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$$

$$\frac{ml\omega^2}{A} \le 48 \times 10^7 \Rightarrow \omega^2 \le \frac{(48 \times 10^7)A}{ml}$$

$$\Rightarrow \omega^2 \le \frac{(48 \times 10^7)(10^{-6})}{10 \times 3} = 16 \Rightarrow \omega_{max} = 4 \text{ rad/s}$$

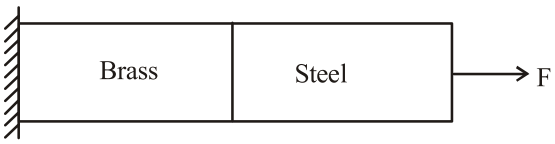
3. **(c)** $\Delta_{temp} = \Delta_{force}$

or $L\alpha(\Delta T) = \frac{FL}{AY}$ $\therefore$ $\alpha = \frac{FL}{AYT} = \frac{F}{\pi r^2 YT}$

Coefficient of volume expression

$$r = 3\alpha = \frac{3F}{\pi r^2 YT}.$$

4. **(Bonus)**



Young modulus, $Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$

Let σ be the stress

Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] \quad [\because L_1 = L_2 = 1\text{m}]$$

$$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$$

$$= 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9 = 8 \times 10^6 \frac{N}{m^2}$$

5. **(b)** Stress $= \frac{F}{A} = \frac{400 \times 4}{\pi d^2} = 379 \times 10^6$ N/m^2

$$\Rightarrow d^2 = \frac{400 \times 4}{379 \times 10^6 \pi}$$

$d = 1.15$ mm

6. **(c)** Given,

Radius of wire, $r = 2$ mm

Mass of the load m = 4 kg

$$\text{Stress} = \frac{F}{A} = \frac{mg}{\pi(r)^2}$$

$$= \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{ N/m}^2$$

7. **(c)** Given,

Radius of wire, $r = 2$ mm

Mass of the load m = 4 kg

$$\text{Stress} = \frac{F}{A} = \frac{mg}{\pi(r)^2} = \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{ N/m}^2$$

8. **(c)** $\Delta_1 = \Delta_2$

or $\frac{Fl_1}{\pi r_1^2 y_1} = \frac{Fl_2}{\pi r_2^2 y_2}$ or $\frac{2}{R^2 \times 7} = \frac{1.5}{2^2 \times 4}$

$\therefore R = 1.75$ mm

9. **(b)** For same material the ratio of stress to strain is same

For first cube

$$\text{Stress}_1 = \frac{\text{force}_1}{\text{area}_1} = \frac{10^5}{(0.1^2)}$$

$$\text{Strain}_1 = \frac{\text{change in length}_1}{\text{original length}_1} = \frac{0.5\times10^{-2}}{0.1}$$

For second block,

$$\text{stress}_2 = \frac{\text{force}_2}{\text{area}_2} = \frac{10^5}{(0.2^2)}$$

$$\text{strain}_2 = \frac{\text{change in length}_2}{\text{original length}_2} = \frac{x}{0.2}$$

x is the displacement for second block.

For same material, $\frac{\text{stress}_1}{\text{strain}_1} = \frac{\text{stress}_2}{\text{strain}_2}$

or, $$\frac{\frac{10.5}{(0.1)^2}}{\frac{0.5\times10^{-2}}{0.1}} = \frac{\frac{10^5}{(0.2)^2}}{\frac{x}{0.2}}$$

Solving we get, $x = 0.25$ cm

10. (a) Young's modulus $Y = \frac{F}{A} / \frac{\Delta\ell}{\ell}$

$$Y = \frac{F\ell}{\pi r^2 \Delta\ell}$$

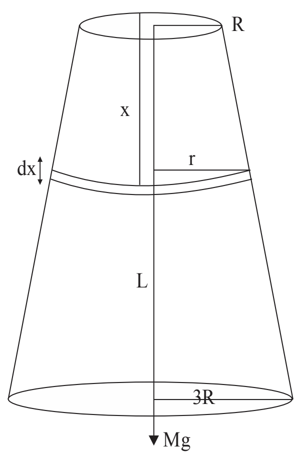
Given, radius r = 5mm, force F = 50πk N,

$$\frac{\ell}{\Delta\ell} = 0.01 \text{ mm}$$

$$\therefore\ Y = \frac{F}{\pi r^2}\frac{\ell}{\Delta\ell} = 2\times10^{14}\,\text{N/m}^2.$$

11. (c) Consider a small element dx of radius r,

$$r = \frac{2R}{L}x + R$$



At equilibrium change in length of the wire

$$\int_0^1 dL = \int \frac{Mg\,dx}{\pi\left[\frac{2R}{L}x + R\right]^2 y}$$

Taking limit from 0 to L

$$\Delta L = \frac{Mg}{\pi y}\left[-\frac{1}{\left[\frac{2Rx}{L}+R\right]_0^L} \times \frac{L}{2R}\right] = \frac{MgL}{3\pi R^2 y}$$

The equilibrium extended length of wire = L + ΔL

$$= L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{1}{3}\frac{Mg}{\pi Y R^2}\right)$$

12. (a) Young's modulus $Y = \frac{\text{stress}}{\text{strain}}$

stress = Y × strain
Stress in steel wire = Applied pressure
Pressure = stress = Y × strain

$$\text{Strain} = \frac{\Delta L}{L} = \alpha \Delta T$$

(As length is constant)
$= 2\times10^{11}\times1.1\times10^{-5}\times100 = 2.2\times10^8$ Pa

13. (b) Tension in the wire, $T = \left(\frac{2mM}{m+M}\right)g$

$$\text{Stress} = \frac{\text{Force / Tension}}{\text{Area}} = \frac{2mM}{A(m+M)}g$$

$$= \frac{2(m\times2m)g}{A(m+2m)} \quad (M = 2\text{ m given})$$

$$= \frac{4m^2}{3mA}g = \frac{4mg}{3A}$$

14. (d) $Y_c \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow 1\times10^{11}\times\left(\frac{1\times10^{-3}}{1}\right) = 2\times10^{11}\times\left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore\ \Delta L_s = \frac{0.5\times10^{-3}}{2} = 0.25\text{ mm}$$

Therefore, total extension of the composite wire
$= \Delta L_c + \Delta L_s$
= 1 mm + 0.25 m = 1.25 m

15. (c) Given, $y = 2\times10^{11}\ Nm^{-2}$

$$\text{Stress}\left(\frac{F}{A}\right) = 5\times10^{7}\ Nm^{-2}$$

$$\Delta V = 0.02\% = 2\times10^{-4}\ m^{3}$$

$$\frac{\Delta r}{r} = ?$$

$$\gamma = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain}\left(\frac{\Delta \ell}{\ell_0}\right) = \frac{\gamma}{\text{stress}} \quad \ldots(i)$$

$$\Delta V = 2\pi r \ell_0 \Delta r - \pi r^2 \Delta \ell \quad \ldots(ii)$$

From eqns (i) and (ii) putting the value of $\Delta\ell$, ℓ_0 and ΔV and solving we get

$$\frac{\Delta r}{r} = 0.25\times10^{-4}$$

16. (c) According to questions,

$$\frac{\ell_s}{\ell_b} = a,\ \frac{r_s}{r_b} = b,\ \frac{y_s}{y_b} = c,\ \frac{\Delta \ell s}{\Delta \ell_b} = ?$$

As, $y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{Ay}$

$$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 . y_s} \quad [\because F_s = (M+2M)g]$$

$$\Delta\ell_b = \frac{2Mg\ell_b}{\pi r_b^2 . y_b} \quad [\because F_b = 2Mg]$$

$$\therefore \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\dfrac{3Mg\ell_s}{\pi r_s^2 . y_s}}{\dfrac{2Mg.\ell_b}{\pi r_b^2 . y_b}} = \frac{3a}{2b^2C}$$

17. (c) Breaking force α area of cross section of wire
Load hold by wire is independent of length of the wire.

18. (a) Given: $F = 100\ kN = 10^5\ N$

$Y = 2\times10^{11}\ Nm^{-2}$

$\ell_0 = 1.0\ m$

radius $r = 10\ mm = 10^{-2}\ m$

From formula, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

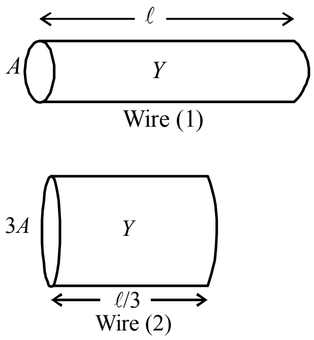
$$\Rightarrow \quad \text{Strain} = \frac{\text{Stress}}{Y} = \frac{F}{AY}$$

$$= \frac{10^5}{\pi r^2 Y} = \frac{10^5}{3.14\times10^{-4}\times2\times10^{11}} = \frac{1}{628}$$

Therefore % strain $= \dfrac{1}{628}\times100 = 0.16\%$

19. (a) From the graph, it is clear that for the same value of load, elongation is maximum for wire *OA*. Hence *OA* is the thinnest wire among the four wires.

20. (c)



For wire 1

Length, $L_1 = 1$

Area, $A_1 = A$

For wire 2

Length, $L_2 = \dfrac{\ell}{3}$

Area, $A_2 = 3A$

As the wires are made of same material, so they will have same young's modulus.

For wire 1,

$$Y = \frac{F/A}{\Delta x/\ell} \quad \ldots(i)$$

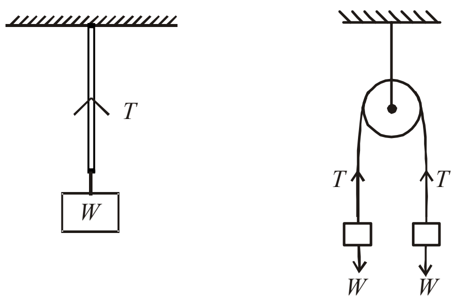
For wire 2,

$$Y = \frac{F'/3A}{\Delta x/(\ell/3)} \quad \ldots(ii)$$

From (i) and (ii) we get,

$$\frac{F}{A}\times\frac{\ell}{\Delta x} = \frac{F'}{3A}\times\frac{\ell}{3\Delta x} \Rightarrow F' = 9F$$

21. (a) Case (i)



At equilibrium, $T = W$

Young's modules, $Y = \dfrac{W/A}{\ell/L}$ .....(1)

Elongation, $\ell = \dfrac{W}{A}\times\dfrac{L}{Y}$

Case (ii) At equilibrium $T = W$

$\therefore$ Young's moduls, $Y = \dfrac{W/A}{\dfrac{\ell/2}{L/2}}$

$\Rightarrow \quad Y = \dfrac{W/A}{\ell/L} \quad \Rightarrow \quad \ell = \dfrac{W}{A} \times \dfrac{L}{Y}$

$\Rightarrow$ Elongation is the same.

22. (a) If force F acts along the length L of the wire of cross-section A, then energy stored in unit volume of wire is given by

Energy density $= \dfrac{1}{2}$ stress × strain

$= \dfrac{1}{2} \times \dfrac{F}{A} \times \dfrac{F}{AY} \left(\because \text{ stress} = \dfrac{F}{A} \text{ and strain} = \dfrac{X}{AY} \right)$

$= \dfrac{1}{2}\dfrac{F^2}{A^2 Y} = \dfrac{1}{2}\dfrac{F^2 \times 16}{(\pi d^2)^2 Y} = \dfrac{1}{2}\dfrac{F^2 \times 16}{\pi d^4 Y}$

If u_1 and u_2 are the densities of two wires, then

$\dfrac{u_1}{u_2} = \left(\dfrac{d_2}{d_1}\right)^4 \Rightarrow \dfrac{d_1}{d_2} = (4)^{1/4} \Rightarrow \dfrac{d_1}{d_2} = \sqrt{2} : 1$

23. (a) When a catapult is stretched up to length l, then the stored energy in it = Δk. E ⇒

$\dfrac{1}{2} \cdot \left(\dfrac{YA}{L}\right)(\Delta l)^2 = \dfrac{1}{2} m v^2 \quad \Rightarrow y = \dfrac{m v^2 L}{\Delta (\Delta l)^2}$

$m = 0.02$ kg
$v = 20\ ms^{-1}$
$L = 0.42$ m
$A = (\pi d^2)/(4)$
$d = 6 \times 10^{-3}$ m
$\Delta l = 0.2$ m

$y = \dfrac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04} = 2.3 \times 10^6\ N/m^2$

So, order is 10^6.

24. (c) Bulk modulus, $K = \dfrac{\text{volumetric stress}}{\text{volumetric strain}}$

$K = \dfrac{mg}{a\left(\dfrac{dV}{V}\right)}$

$\Rightarrow \quad \dfrac{dV}{V} = \dfrac{mg}{Ka}$ ....(i)

volume of sphere, $V = \dfrac{4}{3}\pi R^3$

Fractional change in volume $\dfrac{dV}{V} = \dfrac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\dfrac{3dr}{r} = \dfrac{mg}{Ka}$

$\therefore \dfrac{dr}{r} = \dfrac{mg}{3Ka}$ (fractional decrement in radius)

25. (d) Stress $= \dfrac{\text{Normal force}}{\text{Area}} = \dfrac{N}{A} = \dfrac{N}{(2\pi a)b}$

Stress = B×strain

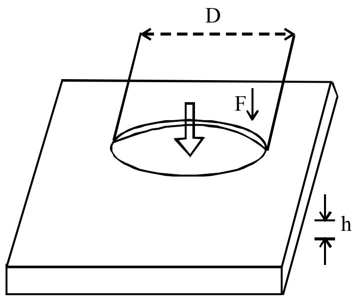
$\dfrac{N}{(2\pi a)b} = B\dfrac{2\pi a \Delta a \times b}{\pi a^2 b}$

$\Rightarrow N = B\dfrac{(2\pi a)^2 \Delta a b^2}{\pi a^2 b}$

Force needed to push the cork.

$f = \mu N = \mu 4\pi b \Delta a B = (4\pi\mu B b)\Delta a$

26. (c)



Shearing strain is created along the side surface of the punched disk. Note that the forces exerted on the disk are exerted along the circumference of the disk, and the total force exerted on its center only.

Let us assume that the shearing stress along the side surface of the disk is uniform, then

$F = \int_{\text{surface}} dF_{max} = \int_{\text{surface}} \sigma_{max} dA = \sigma_{max} \int_{\text{surface}} dA$

$= \int \sigma_{max} \cdot A = \sigma_{max} \cdot 2\pi\left(\dfrac{D}{2}\right)h$

$= 3.5 \times 10^8 \times \left(\dfrac{1}{2} \times 10^{-2}\right) \times 0.3 \times 10^{-2} \times 2\pi$

$= 3.297 \times 10^4 \simeq 3.3 \times 10^4$ N

27. (a) Compressibility $= \dfrac{1}{\text{Bulk modulus}}$

As bulk modulus is least for ethanol (0.9) and maximum for mercury (25) among ehtanol, mercury and water. Hence

compression in volume $\dfrac{\Delta V}{V}$

Ethanol > Water > Mercury

28. **(c)** Poisson's ratio, $\sigma = \frac{\text{lateral strain }(\beta)}{\text{longitudinal strain }(\alpha)}$

For material like copper, $\sigma = 0.33$
And, $Y = 3k(1-2\sigma)$

Also, $\frac{9}{Y} = \frac{1}{k} + \frac{3}{\eta}$

$Y = 2\eta(1+\sigma)$
Hence, $\eta < Y < k$

29. **(a)** Energy stored in the wire per unit volume,

$$E = \frac{1}{2} \times \text{stress} \times \text{strain} \quad ...(i)$$

We know that,

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\Rightarrow \text{strain} = \frac{\text{stress}}{Y}$$

On substituting the expression of strain in equation (i) we get

$$E = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} = \frac{1}{2} \cdot \frac{S^2}{Y}$$

30. **(d)** Let A and L be the area and length of the wire.
Work done by constant force in displacing the wire by a distance ℓ.
= change in potential energy

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\ell}{L} \times A \times L = \frac{F\ell}{2}$$